Color filamentation in ultrarelativistic heavy-ion collisions

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(5-th June 1996)

We study color fluctuations in the quark-gluon plasma produced at the early stage of nucleus-nucleus collision at RHIC or LHC. The fluctuating color current, which flows along the beam, can be very *large* due to the strong anisotropy of the parton momentum distribution. A specific fluctuation, which splits the parton system into the current filaments parallel to the beam direction, is argued to grow exponentially. The physical mechanism responsible for the phenomenon, which is known as a filamentation instability, is discussed.

In the near future the nucleus-nucleus collisions will be studied experimentally at the accelerators of a new generation: Relativistic Heavy-Ion Collider (RHIC) at Brookhaven and Large Hadron Collider (LHC) at CERN. The collision energy will be larger by one or even several orders of magnitude than that one of the currently operating machines. A copious production of partons, mainly gluons, due to hard and semihard processes is expected in the heavy-ion collisions at this new energy domain [1]. Thus, one deals with the many-parton system at the early stage of the collision. The system is on average locally colorless but random fluctuations can break the neutrality. Since the system is initially far from equilibrium the color fluctuations can noticeably influence its evolution.

In our previous papers [2] we have studied the plasma oscillations in the nonequilibrium quark-gluon system produced in ultrarelativistic heavy-ion collisions. We have argued that due to the strong anisotropy of the parton momentum distribution there are unstable modes which exponentially grow in time. Here we show that the fluctuations which initiate these modes are *large*, much larger than in the equilibrium plasma. We also discuss the physical mechanism responsible for the growth of the fluctuation which splits the parton system into the color current filaments parallel to the beam direction. For completeness we recapitulate at the end of this letter the results from our papers [2], which show the existence of the unstable modes.

The distribution functions of quarks $Q_{ij}(t,\mathbf{x},\mathbf{p})$, antiquarks $\bar{Q}_{ij}(t,\mathbf{x},\mathbf{p})$, and gluons $G_{ab}(t,\mathbf{x},\mathbf{p})$ with $i,j=1,2,3$ and $a,b=1,2,...,8$ are [3,4] matrices in the color space; t, $\mathbf{x}$ and $\mathbf{p}$ denote the time, position, and the momentum. The color current expressed through these functions reads [3,4]

$$j^\mu_a(t,\mathbf{x}) = \frac{1}{2}\, g \int \frac{d^3p}{(2\pi)^3}\, \frac{p^\mu}{E_p} \left(\tau^a_{ji} \Big(Q_{ij}(t,\mathbf{x},\mathbf{p}) - \bar{Q}_{ij}(t,\mathbf{x},\mathbf{p}) \Big) + i f^{abc} G_{bc}(t,\mathbf{x},\mathbf{p}) \right) ,$$

where g is the QCD coupling constant, τ^a is the $SU(3)$ group generator, f^{abc} the respective structure constant, and $p^\mu = (E_p, \mathbf{p})$ is the parton four-momentum with $E_p = |\mathbf{p}|$ being the energy of the massless quark or gluon.

We assume that the quark-gluon plasma is on average locally colorless, homogeneous, and stationary. Thus, the distribution functions averaged over ensemble are of the form

$$\langle Q_{ij}(t,\mathbf{x},\mathbf{p})\rangle = \delta_{ij} n(\mathbf{p}) \;, \qquad \langle \bar{Q}_{ij}(t,\mathbf{x},\mathbf{p})\rangle = \delta_{ij} \bar{n}(\mathbf{p}) \;, \qquad \langle G_{ij}(t,\mathbf{x},\mathbf{p})\rangle = \delta_{ij} n_g(\mathbf{p}) \;,$$

which give the zero average color current.

We find the fluctuations of the color current generalizing a well-known formula for the electric current [5]. For a system of noninteracting quarks and gluons we have in the classical limit the following expression

$$M^{\mu\nu}_{ab}(t,\mathbf{x}) \stackrel{\rm def}{=} \langle j^\mu_a(t_1,\mathbf{x}_1) j^\nu_b(t_2,\mathbf{x}_2)\rangle = \frac{1}{8}\, g^2\, \delta^{ab} \int \frac{d^3p}{(2\pi)^3}\, \frac{p^\mu p^\nu}{E_p^2}\, f(\mathbf{p})\, \delta^{(3)}(\mathbf{x} - \mathbf{v}t) \;, \tag{1}$$

where the effective parton distribution function $f(\mathbf{p})$ equals $n(\mathbf{p}) + \bar{n}(\mathbf{p}) + 6 n_g(\mathbf{p})$, $(t,\mathbf{x}) \equiv (t_2 - t_1, \mathbf{x}_2 - \mathbf{x}_1)$, and $\mathbf{v} = \mathbf{p}/E_p$ is the parton velocity. Due to the average space-time homogeneity the correlation tensor depends only on the difference $(t_2 - t_1, \mathbf{x}_2 - \mathbf{x}_1)$.

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The physical meaning of the formula (1) is transparent. The space-time points $(t_1, \mathbf{x}_1)$ and $(t_2, \mathbf{x}_2)$ are correlated in the system of noninteracting particles if the particles fly from $(t_1, \mathbf{x}_1)$ to $(t_2, \mathbf{x}_2)$. Therefore the delta $\delta^{(3)}(\mathbf{x} - \mathbf{v}t)$ is present in the formula (1). The momentum integral of the distribution function simply represents the summation over particles.

One finds the Fourier spectrum of the fluctuations from eq. (1) as

$$M^{\mu\nu}_{ab}(\omega, \mathbf{k}) = \frac{1}{8}\, g^2\, \delta^{ab} \int \frac{d^3p}{(2\pi)^3}\, \frac{p^\mu p^\nu}{E_p^2}\, f(\mathbf{p})\, 2\pi\delta(\omega - \mathbf{k}\mathbf{v}) \;, \tag{2}$$

where ω is the frequency and $\mathbf{k}$ the wave-vector.

We model the parton momentum distribution at the early stage of ultrarelativistic heavy-ion collision in two ways:

$$f(\mathbf{p}) = \frac{1}{2Y} \Theta(Y - y)\Theta(Y + y)\; h(p_\perp)\; \frac{1}{p_\perp\, \mathrm{ch} y} \;, \tag{3}$$

and

$$f(\mathbf{p}) = \frac{1}{2\mathcal{P}} \Theta(\mathcal{P} - p_\parallel)\Theta(\mathcal{P} + p_\parallel)\; h(p_\perp) \;, \tag{4}$$

where y, $p_\parallel$, $p_\perp$, y, and ϕ denote the parton rapidity, the longitudinal and transverse momenta, and the azimuthal angle, respectively. The parton momentum distribution (3) corresponds to the rapidity distribution which is flat in the interval $(-Y, Y)$. The distribution (4) is flat for the longitudinal momentum $-\mathcal{P} < p_\parallel < \mathcal{P}$. We do not specify the transverse momentum distribution $h(p_\perp)$, which is assumed to be of the same shape for quarks and gluons, because it is sufficient for our considerations to demand that the distributions (3, 4), are strongly elongated along the $z-$axis i.e. $e^Y \gg 1$ and $\langle p_\parallel \rangle \gg \langle p_\perp \rangle$.

Due to the symmetry $f(\mathbf{p}) = f(-\mathbf{p})$ of the distributions (3,4), the correlation tensor $M^{\mu\nu}$ is diagonal i.e. $M^{\mu\nu} = 0$ for $\mu \neq \nu$. Since the average parton longitudinal momentum is much bigger than the transverse one, it obviously follows from eq. (2) that the largest fluctuating current appears along the $z-$axis. Therefore, we discuss the M^{zz} component of the correlation tensor. $M^{zz}(\omega, \mathbf{k})$ depends on the $\mathbf{k}-$vector orientation and there are two generic cases: $\mathbf{k} = (k_x, 0, 0)$ and $\mathbf{k} = (0, 0, k_z)$. The inspection of eq. (2) shows that the fluctuations with $\mathbf{k} = (k_x, 0, 0)$ are much larger than those with $\mathbf{k} = (0, 0, k_z)$. Thus, we compute $M^{zz}(\omega, k_x)$.

Substituting the distributions (3,4) into (2) one finds after azimuthal integration

$$M^{zz}_{ab}(\omega, k_x) = \delta^{ab}\, \frac{g^2}{32\pi^2\, Y} \int_{-Y}^{Y} dy \int_0^\infty dp_\perp\; h(p_\perp)\, p_\perp\, \frac{\mathrm{sh}^2 y}{\mathrm{ch} y}\, \frac{\Theta(k_x^2 - \omega^2\, \mathrm{ch}^2 y)}{\sqrt{k_x^2 - \omega^2\, \mathrm{ch}^2 y}} \;, \tag{5}$$

$$M^{zz}_{ab}(\omega, k_x) = \delta^{ab}\, \frac{g^2}{32\pi^2 \mathcal{P}} \int_{-\mathcal{P}}^{\mathcal{P}} dp_\parallel \int_0^\infty dp_\perp\; h(p_\perp)\, \frac{p_\perp p_\parallel^2}{E_p}\, \frac{\Theta(k_x^2 p_\perp^2 - \omega^2 E_p^2)}{\sqrt{k_x^2 p_\perp^2 - \omega^2 E_p^2}} \;. \tag{6}$$

One observes that the integrals from eqs. (5,6) reach the maximal values for $\omega^2 \ll k_x^2$. Therefore, we compue M^{zz}_{ab} for $\omega = 0$. Keeping in mind that $e^Y \gg 1$ and $\langle p_\parallel \rangle \gg \langle p_\perp \rangle$ we get the following approximate expressions for the flat $y-$ and $p_\parallel-$distributions:

$$M^{zz}_{ab}(\omega = 0, k_x) = \frac{1}{8}\, g^2\, \delta^{ab}\, \frac{e^Y}{Y}\, \frac{\langle \rho \rangle}{|k_x|} \;, \tag{7}$$

$$M^{zz}_{ab}(\omega = 0, k_x) = \frac{1}{8}\, g^2\, \delta^{ab}\, \frac{\mathcal{P}}{\langle p_\perp \rangle}\, \frac{\langle \rho \rangle}{|k_x|} \;, \tag{8}$$

where $\langle \rho \rangle$ is the effective parton density given as

$$\langle \rho \rangle \equiv \int \frac{d^3p}{(2\pi)^3}\, f(\mathbf{p}) = \frac{1}{4\pi^2} \int_0^\infty dp_\perp p_\perp\; h(p_\perp) = \frac{1}{3}\, \langle \rho \rangle_{q\bar{q}} + \frac{3}{4}\, \langle \rho \rangle_g \;,$$

with $\langle \rho \rangle_{q\bar{q}}$ denoting the average density of quarks and antiquarks, and $\langle \rho \rangle_g$ that of gluons. For the flat $p_\parallel-$case we have also used the approximate equality

$$\int_0^\infty dp_\perp \; h(p_\perp) = \frac{1}{\langle p_\perp \rangle} \int_0^\infty dp_\perp p_\perp \; h(p_\perp)$$

to get the expression (8)

It is instructive to compare the results (7, 8) with the analogous one for the equilibrium plasma which is

$$M^{zz}_{ab}(\omega = 0, k_x) = \frac{\pi}{16}\, g^2 \, \delta^{ab} \, \frac{\langle \rho \rangle}{|k_x|} \; .$$

One sees that *the current fluctuations in the anisotropic plasma are amplified by the large factor* which is e^Y/Y or $\mathcal{P}/\langle p_\perp \rangle$.

Let us now discuss how the fluctuation, which contributes to $M^{zz}_{ab}(\omega = 0, k_x)$, evolves in time. The form of the fluctuating current is

$$\mathbf{j}_a(x) = j_a \, \hat{\mathbf{e}}_z \cos(k_x x) \; , \tag{9}$$

where $\hat{\mathbf{e}}_z$ is the unit vector in the $z-$direction. Thus, there are current filaments of the thickness $\pi/|k_x|$ with the current flowing in the opposite directions in the neighboring filaments.

In the limit of weak fields the chromodynamics can be approximately treated as an eight-fold electrodynamics. Consequently, the magnetic field generated by the current (9) is given as

$$\mathbf{B}_a(x) = \frac{j_a}{k_x} \, \hat{\mathbf{e}}_y \sin(k_x x) \; .$$

The Lorentz force acting on the partons, which fly along the beam, equals

$$\mathbf{F}(x) = q_a \, \mathbf{v} \times \mathbf{B}_a(x) = -q_a \, v_z \, \frac{j_a}{k_x} \, \hat{\mathbf{e}}_x \sin(k_x x) \; ,$$

where q_a is the color charge. One observes, see Fig. 1, that the force distributes the partons in such a way that those, which positively contribute to the current in a given filament, are focused to the filament center while those, which negatively contribute, are moved to the neighboring one. Thus, the initial current is growing.

The mechanism described here is well-known in the plasma physics [7] and it leads to the so-called filamentation or Weibel instability [6]. In the context of the quark-gluon plasma the phenomenon has been first discussed in the system of two interpenetrating parton beams [8]. Such a system however seems to be completely unrealistic from the experimental point of view. Then it has been argued [2] that the filamentation instability can occur under the conditions which will be realized in heavy-ion collisions at RHIC and LHC. Let us briefly recapitulate these considerations which confirm the qualitative arguments presented above.

The spectrum of plasma modes initiated by the color fluctuations is determined by the dispersion equation which for the anisotropic plasma is

$$\det|\mathbf{k}^2\delta^{ij} - k^i k^j - \omega^2 \epsilon^{ij}(\omega, \mathbf{k})| = 0 \; , \qquad i,j = x,y,z$$

where the chromodielectric tensor ϵ^{ij} is

$$\epsilon^{ij}(\omega, \mathbf{k}) = \delta^{ij} + \frac{g^2}{2\omega} \int \frac{d^3p}{(2\pi)^3} \frac{v^i}{\omega - \mathbf{kv} + i0^+} \frac{\partial f(\mathbf{p})}{\partial p^l} \left[\Big(1 - \frac{\mathbf{kv}}{\omega}\Big)\delta^{lj} + \frac{k^l v^j}{\omega} \right] .$$

For the specific color fluctuation with wave vector along the $x-$axis and the electric field parallel to the $z-$direction, the dispersion equation simplifies to

$$H(\omega) \equiv k_x^2 - \omega^2 \epsilon^{zz}(\omega, k_x) = 0 \; . \tag{10}$$

The so-called Penrose criterion states that *the dispersion equation* $H(\omega) = 0$ *has unstable solutions if* $H(\omega = 0) < 0$. We have shown that under the reasonable assumptions concerning the form of the transverse momentum distribution $h(p_\perp)$, the criterion is indeed satisfied. Then, we have solved approximately the dispersion equation (10) and found the unstable mode with the pure imaginary frequency. The estimated characteristic time of the instability development, which appears as short as $0.3 - 0.4$ fm$/c$, is significantly smaller than the life time of the plasma created in the nuclear collision. Therefore, the instability can indeed develop and play a significant role in the system dynamics.

One asks whether the color instabilities are detectable in ultrarelativistic heavy-ion collisions. The answer seems to be positive because the occurrence of the filamentation breaks the azimuthal symmetry of the system and hopefully will be visible in the final state. The azimuthal orientation of the wave vector will change from one collision to another while the instability growth will lead to the energy transport along this vector (the Poynting vector points in this direction). Consequently, one expects significant variation of the transverse energy as a function of the azimuthal angle. This expectation is qualitatively different than that based on the parton cascade simulations [1], where the fluctuations are strongly damped due to the large number of uncorrelated partons. Due to the collective character of the filamentation instability the azimuthal symmetry will be presumably broken by a flow of large number of particles with relatively small transverse momenta. The jets produced in hard parton-parton interactions also break the azimuthal symmetry. In this case however the symmetry is broken due to a few particles with large transverse momentum. The problem obviously needs further studies but it seems that the event-by-event analysis of the nuclear collision give a chance to observe the color instabilities in the experiments planed at RHIC and LHC.

This work was partially supported by the Polish Committee of Scientific Research under Grant No. 2 P03B 195 09.

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Figure captions

FIG. 1. The mechanism of filamentation. The phenomenon is, for simplicity, considered in terms of the electrodynamics. The fluctuating current generates the magnetic field acting on the positively charged particles which in turn contribute to the current (see text). $\otimes$ and $\odot$ denote the parallel and, respectively, antiparallel orientation of the magnetic field with respect to the $y-$axis.